MATH 54 - HINTS TO HOMEWORK 3

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Here are a couple of hints to Homework 3! Enjoy! :)

SECTION 1.10: LINEAR MODELS IN BUSINESS, SCIENCE, AND ENGINEERING

You can ignore this section if you want, it's for your own personal entertainment :)

SECTION 2.1: MATRIX OPERATIONS

Remember the rule $(m \times n) \bullet (n \times p) = (m \times p)$.

2.1.9. Write out AB and BA and solve for k in $AB = BA$.

2.1.11. $D = 2I$ works!

2.1.13. $[Q\mathbf{r_1} \cdots Q\mathbf{r_p}] = Q\,[\mathbf{r_1} \cdots \mathbf{r_p}]$. Think of this in terms of how you multiply matrices! **Remember this for later on, it will be useful!**

2.1.15.

(a) **F** (oh, life would be awesome if this was true! But $\mathbf{a_1}\mathbf{a_2}$ doesn't even make sense!)
(b) **F** (the columns of **A** using weights from the column of **B**)
(c) **T**
(d) **T**
(e) **F** (in the *reverse* order, $(AB)^T = B^T A^T$)

2.1.17. This amounts to solving a bunch of equations! Label the entries in the first two columns of B as a, b, c, d, and solve for a, b, c, d using the fact that AB is given.

2.1.23. Multiply the equation $A\mathbf{x} = \mathbf{0}$ by C.

2.1.24. $D\mathbf{b}$ is a solution!

SECTION 2.2: THE INVERSE OF A MATRIX

2.2.1, 2.2.3. Use theorem 4.

2.2.9. All statements are true, except for (b), because $(AB)^{-1} = B^{-1}A^{-1}$.

2.2.11. Muliply $AX = B$ by A^{-1} and use $A^{-1}A = I$.

2.2.19. $X = CB - A$

2.2.21. In other words, you have to show that the only solution to $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$ (think about this in terms of linear combinations of the columns of A). For that, multiply the equation $A\mathbf{x} = \mathbf{0}$ by A^{-1}.

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2.2.38.

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The best way to get D is by using the equation $AD = I_2$ and guessing!

C cannot exist, because otherwise A would be invertible, and in particular its columns would be linearly independent, which is bogus!

SECTION 2.3: CHARACTERIZATIONS OF INVERTIBLE MATRICES

For the first few problems, row-reduction is the key!

2.3.11. Just look at theorem 8! If one of those statements holds, then all of them hold!

2.3.13. Only if all the entries on the diagonal are nonzero! See theorem $8(c)$

2.3.15. No! See theorem $8(e)$

2.3.17. This is super cute!!! If A is invertible, then A^{-1} is invertible (because $\left(A^{-1}\right)^{-1} = A$). Hence theorem $8(e)$ holds for A^{-1} instead of A).

2.3.21. **Careful!** The word *some* is key here! (if the statement held for *all* $\mathbf{y}$, then it would be true). In this case G is not invertible, and hence by Theorem $8(h)$, the columns cannot span the whole space.

2.3.24. Yes, because L is invertibel, and hence Theorem $8(h)$ holds!

2.3.30. It is invertible by theorem $8(f)$ and hence also onto by theorem $8(i)$. This is one of the special features of $\mathbb{R}^n$ and about linear transformations! You cannot expect this result to be true in general!

SECTION 2.5: MATRIX FACTORIZATIONS

You should be able to do this section without any help! :)

SECTION 2.8: SUBSPACES OF $\mathbb{R}^n$

2.8.1. Not closed under scalar multiplication

2.8.3. Not closed under addition

2.8.5. One way to do this is to group all the three vectors together in a matrix A, and solve $A\mathbf{x} = \mathbf{0}$.

2.8.7.

(a) 3
(b) Infinitely many of them! (but in a sense, you'll see that $Col(A)$ is a 2 or 3 dimensional space).
(c) Solve $A\mathbf{x} = \mathbf{p}$

2.8.9. Use the row-reduced form of A.

2.8.11. q is the number of pivots, $q = n - p =$ number of free variables

2.8.21. All statements are **True**, **EXCEPT** (c) (should be $\mathbb{R}^n$)! Notice that in particular (a) is true! The book is just being anal about this, even though it omitted the word 'for each', the statement still remains true (the words 'for each' here are implied)

2.8.23. To find $Nul(A)$, solve $A\mathbf{x} = \mathbf{0}$ using the row-echelon form. To find $Col(A)$, notice that the first two columns of A are pivot columns. In particular, a basis for $Col(A)$ is the set of the first two columns of the *original* matrix A.

SECTION 2.9: DIMENSION AND RANK

2.9.1. $[\mathbf{x}]_{\mathcal{B}}$ gives you the coefficients of the linear combination of $\mathbf{x}$ in the basis $\mathcal{B}$.

2.9.3. Find the coefficients of $\mathbf{x}$ as a linear combination of $\mathbf{b_1}$ and $\mathbf{b_2}$.

2.9.9, 2.9.11. To find $Nul(A)$, solve $A\mathbf{x} = \mathbf{0}$ using the row-echelon form. To find $Col(A)$, locate the pivot columns of A. In particular, a basis for $Col(A)$ is the set of the pivot columns of the *original* matrix A.

2.9.17. All statements are **TRUE**, **EXCEPT** for (b) (the line must go through the origin!)

2.9.21. Use the fact that dim($Nul(A)$) + rank(A) $= n$

2.9.23. For example:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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